MAG02 primes

Nick Smith

Prime Numbers, Factors and Fractions

The aim of this lesson is to enable you to:

- work with prime numbers, factors, multiples and sequences
- add, subtract, multiply and divide fractions
- calculate equivalent fractions
- use inverse operations

In the last lesson we looked at the number system in general. Now we need to consider different kinds of number

in a bit more detail. Prime numbers and fractions, in particular, will be relevant throughout your GCSE work.

Prime Numbers

What do we call a number that can only be divided by itself and by 1?

It is called a **prime** number. It would appear as if 1 itself would fit this definition, but, by convention, 1 is **not** taken as prime, so that the prime numbers, in sequence, are: 2, 3, 5, 7, 11, 13, …

There are some points to note here. The prime numbers are always *whole* numbers. 2 is the only even prime number. Obviously, not all odd numbers are prime, e.g. 15 is not, since 15 can be divided by 3 and 5, as well as itself and 1.

There is no easy pattern in the order of the prime numbers. For instance, they do not always go up 2 at a time nor are there the same number of primes in each group of 10 whole numbers. For instance, between 30 and 40, there are only two primes, i.e. 31 and 37, but between 40 and 50 there are three primes, i.e. 41, 43 and 47. But in general, as numbers increase, primes become steadily less frequent.

How do we know whether a number is a prime number or not? A number is prime if it is only divisible by 1 and itself. To determine whether a number is prime or not, try dividing by smaller numbers. It is usually necessary to try dividing by a few numbers only.

It is helpful to remember the prime numbers below 20:

2, 3, 5, 7, 11, 13, 17, 19, …

A knowledge of the early square numbers also helps:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100...

Example 1
Is 41 prime?

Look at the list of square numbers: is the last which is less than 41. We only need to look up to 6 for divisors of 41. Furthermore, we do not need to try **all** the numbers up to 6. In fact, we only need to try the **prime** numbers: 2, 3 and 5. None of these divides into 41, so we can safely say that 41 is a prime number.

Think of it this way: if any number up to 35 is the product of two numbers (i.e. it is not prime), then the smaller of those two numbers *must* be less than 6 and we only need to find *one* such factor, e.g. 35 is $\mathbf{5} \times 7$, to prove a number is *not* prime. Similarly, any non-prime ("composite") number up to 48 *must* have at least one lower factor no higher than 6.

Example 2
Is 91 prime?

Look at the list of square numbers: the biggest which is smaller than 91 is . So we only need to try dividing by prime numbers up to 9: these are 2, 3, 5 and 7.

It turns out that 7 does divide into 91: .
So 91 is not a prime number.

Factors and Multiples

A **factor** of a given number is any number that will divide exactly into the given number, e.g. 3 is a factor of 12, since $12 \div 3 = 4$. Note that here we are considering the natural numbers only. It is clear that all numbers have the factors 1 and the number itself; if these are the only factors, then the number is prime. Otherwise, the factors of the given number can be listed in order. So, the factors of 12 are 1, 2, 3, 4, 6 and 12, and those of 25 are 1, 5 and 25.

Example 1
(a) Find all the factors of 12
(b) Find the prime factors of 12.

1 and 12 are factors, so write them with a large gap in between: Try 2: , so 2 and 6 are both factors. Add 2 and 6 to the list, the 2 after 1, and the 6 before 12:	

Try 3: this also divides into 12 4 times, so add 3 and 4 to the list.

All factors are now found. The two halves of the list (‘low’ and ‘high’ factors) have now met in the middle.

1					12
1	2			6	12
1	2	3	4	6	12

Having found all the factors of 12, it is possible to find the prime factors of 12. The prime factors are those factors of 12 which are also prime numbers. In this case, the only prime numbers in the list are 2 and 3.

The **prime factorisation** of 12 is or, using index notation (to be studied later in the course), .

A method for obtaining the prime factorisation of a number is as follows. The overall strategy is to find:

- Which prime numbers divide into the number?
- How many times each of the above prime numbers divide into the number?

Example 2
Find the prime factorisation of 84.

It is usual to show the working as follows: the prime factors appear in the left hand column. The starting number, in this case 84, appears is at the top of the right hand column. The numbers in the right hand column decrease as they are divided by each of the prime numbers in the left hand column. The process stops when the right hand column becomes 1. The prime factorisation is then the product of all the numbers in the left hand column.

2
3
7
84
42
21
7
1

The first prime number is 2. This divides into 84: 84 2 = 42.

2 divides into 42: 42 2 = 21. 2 will not divide in again.

So try the next prime number, 3. This works. 21 3 = 7.

Obviously, the prime number 7 divides into 7.

The process stops now that 1 is reached.

The prime factorisation is now obtained by reading down the left hand column. The prime factorisation of 84 can therefore be written either as or, in index notation, as .

This factorisation is unique (there is only one set of prime numbers that works). This idea is so important it has a theorem: **The Unique Factorisation Theorem**. There is a very similar theorem called **The Fundamental Theorem of Arithmetic** which states that every integer greater than 1 is either prime or can be written as a unique product of primes. Unique factorisation is used in cryptography (the study of secret codes) because it takes even very powerful computers a long time to factor large numbers. Difficult to break codes are vital to the safe use of the internet, online banking and so on.

Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

The HCF can be done mentally, certainly for small numbers. However, for larger numbers it is safer to have a method. This can be summarised as follows:

1. For each number make a list all its factors.
2. Identify the ‘common factors’: these are all the factors which are common to both/all lists.

3. The HCF is the largest of these common factors.

Example 1
Find the HCF of: (a) 56 and 70 (b) 55, 88 and 121.

(a) List all the factors of 56: 1 2 4 7 8 14 28 56
List all the factors of 70: 1 2 5 7 10 14 35 70
List the factors which are in both lists: **1 2 7 14**
The HCF of 56 and 70 is therefore **14**.

(b) List all the factors of 55: 1 5 11 55
List all the factors of 88: 1 2 4 8 11 22 44 88
List all the factors of 121: 1 11 121
List all the factors which are in all three lists: **1 11**
The HCF of 55, 88 and 121 is therefore **11**.

The Least Common Multiple can also be found 'mentally', certainly for small numbers. However, there is a method which ensures a correct answer. This method consists of listing the multiples of each number until a common multiple is found.

Example 2
Find the LCM of: (a) 4 and 5 (b) 4, 6 and 9.

(a) List the multiples of 4 (i.e. starting off with the four times table):
4, 8, 12, 16, 20, 24, 28, …

List the multiples of 5: 5, 10, 15, 20, 25, 30, …

We see that 20 is in both lists. Moreover, it is the smallest number in both lists. The Least Common Multiple (LCM) of 4 and 5 is therefore **20**.

(b) List the multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40…
List the multiples of 6: 6, 12, 18, 24, 30, 36, 42 …
List the multiples of 9: 9, 18, 27, 36, 45, 54, 63…

We see that 36 is in all three lists, and is the smallest such number. So the LCM of 4, 6 and 9 is **36**.

One practical question arises: how do we know how many multiples to list?

We just have to continue the two (or three or…) lists until a common multiple appears. This raises another question: can we be sure that a common multiple will ever appear? In fact, a common multiple must always exist: just multiply the original numbers together.

Thus in example (a) above, is a common multiple. In this case, it is also the Lowest Common Multiple. However, this does not always happen. Thus in Example (b) above, . This is indeed a 'common multiple' of 4, 6 and 9, but it is not the Lowest Common Multiple: we already know that the LCM is 36.

1. Write down all the square numbers from 100 to 200.

•

Start like this: Write your answer here or below

2. List the prime numbers between 50 and 70.

3. Find the HCF of

(a) 9 and 12,
(b) 16 and 35,
(c) 8, 40 and 56.

4. Find the LCM of

(a) 8 and 12,
(b) 10 and 25,
(c) 4, 7 and 21.

5. Find all the factors of: (a) 45 (b) 46 (c) 47
6. Find the next prime number after 31.

7. Find the next two prime numbers after 53.

8. Given the list of numbers: 13 21 22 29 45 47 49, find each of the following:

a) an even number
b) a prime number
c) a multiple of 9
d) a factor of 63
e) a square number.

9. Obtain the prime factorisation of: (a) 24 (b) 42 (c) 60

Language Check!

You will have noticed that many different words can be used to mean the same thing. Use the following table to check that all the standard words used for arithmetic are familiar to you.

Mathematical sign	*Different words with the same meaning*	*Different expressions with the same meaning*
+	add sum total plus	add *a* and *b* *a* plus *b* the sum of *a* and *b* the total of *a* and *b*
−	minus subtract take away difference	*a* minus *b* take *b* away from *a* the difference between *a* and *b*
	multiply times product	*ab* *a* multiplied by *b* *a* times *b* the product of *a* and *b*
÷	divide over quotient	*a* divided by *b* *a* over *b* the quotient of *a* and *b*

Order of Arithmetic Processes

The order in which the arithmetic processes of addition, subtraction, multiplication and division are performed is most important, and certain rules have to be observed.

For two numbers either added or multiplied, the order does not matter,

e.g. $3 + 5 = 8$ and $5 + 3 = 8$; $3 \times 5 = 15$ and $5 \times 3 = 15$.

We say that addition and multiplication are **commutative.** However, if the two numbers are subtracted or divided, the order is vital,

e.g. $8 – 2 = 6$ but $2 – 8 = –6$; $8 \div 2 = 4$ but $2 \div 8 = 1/4$.

So we can see that subtraction and division are **non-commutative.**

With more complicated sums to perform, there are ways to remember what to do first. The word BODMAS is used to give the order: B = brackets, O = of, D = division, M = multiplication, A = addition and S = subtraction.

Example 1
Simplify the expression $7 \times 3 – 32 \div 8$.

Step 1. Put brackets around the division and multiplication (BO<u>DM</u>AS):

$7 \times 3 – 32 \div 8 =$ $(7 \times 3) – (32 \div 8)$

Step 2. Work out the part in brackets:

$= 21 – 4$

Step 3. Do the final subtraction (BODM<u>AS</u>):

$= 17$

Sometimes the BODMAS rule alone is not enough. For example, if you consider $120 \div 6 \times 4$, it is not clear whether the division or multiplication comes first, and brackets would be inserted to make it obvious:

$(120 \div 6) \times 4$ (i)

or $120 \div (6 \times 4)$ (ii)

Note that these do not give the same answer:
(i) gives $20 \times 4 = 80$, but (ii) gives $120 \div 24 = 5$.

Perhaps BODMAS is a bit misleading? In fact, division and multiplication have equal priority. Also, addition and subtraction have equal priority. Here is the rule in simpler form:

- Brackets first
- Division and/or multiplication next
- Finally addition and/or subtraction.

If two operations in an expression appear to have equal priority, then we work from left to right.

Example 2

Do multiplication and division before subtraction:	
Now do subtraction:	
$21 - 4$	
17	

Example 3

Division and multiplication have equal priority, so in this case we must work left to right, and do division first:	
Finally, do the multiplication:	
80	

Example 4

Do brackets first (i.e. multiplication):	
Finally, do the division:	
5	

Note the subtle difference between Examples 3 and 4. The different answer in Example 4 is due to the brackets.

Example 5

This example looks like a fraction. Simplify the top and bottom using the usual BODMAS rules, then cancel the fraction into its lowest terms.

Do brackets first:	

The 3^2 is multiplication in disguise since $3^2 = 3 \times 3$:

Next comes the addition:

Finally, simplify the fraction:

Calculators and BODMAS

Proper calculators should perform BODMAS automatically. Standard scientific calculators such as Casio, Sharp and Texas Instruments are all safe. However, some other calculators, particularly those given away as free gifts, do not seem to know about BODMAS.

A simple test is to try . BODMAS is clear: the multiplication must be done before the addition, so that the answer should be 14. If a calculator gives a result of 20, then it does not know about BODMAS: it has simply worked from left to right. We only work from left to right if operations have **equal priority**.

Use the BODMAS rule to work out the value of these expressions:

-

(a) $3 + 2 \times 8$

(b) $4 \times 5 - 3$

(c) $9 \times 2 + 64 \div 8$

(d) 25 ÷ 5 – 9 ÷ 3

(e) 81 ÷ (5 + 4) – 2 × 3 + 14 ÷ 7

(f) 17 + 9 –14 + 5 – 20 + 1 – 2

(g)

Sequences

When a group of numbers are related to each other by a rule, the group is called a sequence. Some sequences are easy to recognise. For example:

(a) 2, 4, 6, 8, 10

Each term is formed by adding 2 to the previous term.

(b) 6, 3, 0, –3, –6

Each term is formed by subtracting 3 from the previous term.

(c) 1, 4, 9, 16, 25

Each term is a square of the sequence of integers (1, 2, 3, 4, 5, etc.).

Sometimes it is not so easy to see how the sequence has been produced. For example:

1, 5, 12, 22, 35, …

There does not seem to be a single number that can be added or multiplied to produce the sequence.

We then use the method of differences. To use this method, we write down the sequence with spaces between the numbers. We then write down the differences between successive terms. This is the first difference line. If this does not show a pattern, then we repeat the procedure to give a second difference line.

Sequence

1

5

12

22

35

1st difference

4

7

10

13

2nd difference

3

3

3

We can now put in the next two numbers in the 1st difference line. These are 16 and 19. Finally we have the next two terms in the sequence, which are

35 + 16 = 51

and 51 + 19 = 70.

Helpful Hint

Here are some of the sorts of pattern to look out for:

Adding e.g. 5, 8, 11, 14 (adding 3)

Subtracting e.g. 999, 995, 991 (subtracting 4)

Multiplying e.g. 10, 100, 1000, 10000 (multiplying by 10)

Dividing e.g. 64, 32, 16, 8, 4 (dividing by 2)

Squaring e.g. 1, 9, 25, 49 (squares of odd numbers)

Square roots e.g. 256, 16, 4, 2 (each term is the square root of the previous term).

1. Find the next two terms in each of the following sequences:

•

(a) 6, 8, 10, 12, 14,…
(b) 0, 9, 18, 27, 36,…
(c) -14, -10, -6, -2,…
(d) 20, 17, 14, 11, 8,…
(e) 10, 20, 40, 80, 160,…
(f) 100000, 20000, 4000, 800,…
(g) 2, 2, 4, 6, 10, 16, 26,…
(h) 1, 2, 3, 5, 8, 13, 21,…

2 Find the missing terms, denoted by [ ] , in each of the following sequences:

(a) 7, 10, 13, 16, [ ] , 22, 25,…
(b) -22, -14, [ ] , 2, 10, 18,…
(c) 61, [ ] , 49, 43, 37, 31,
(d) 128, 64, 32, [ ] , 8, 4,…
(e) 1, 3, 9, 27, [ ] , 243, 729,…
(f) 3, 5, 8, [ ] , 21, 34, 55,…

3 (a) Write down the first ten square numbers.

(b) Hence find the next two terms in each of the following sequences:

(i) 2, 5, 10, 17, 26, 37, 50, 65,…
(ii) 10, 40, 90, 160, 250, 360, 490, 640,…
(iii) 0, 3, 8, 15, 24, 35, 48, 63,…

4. Use the ‘table of differences’ method to find the next two terms in the following sequences:

(a) 0, 1, 6, 15, 28,…
(b) 2, 2, 5, 11, 20, 32,…

Fractions

It is important to feel comfortable about using fractions since they come up in lots of different areas of mathematics – and in life!

The top of a fraction is called the numerator, the bottom is called the denominator. The denominator tells us how many parts to split something into. The numerator tells us how many parts to take. Thus the fraction means that something is split into three parts, and then two of the parts are taken.

A **vulgar** fraction is a number written by numerator and denominator rather than as a decimal (e.g. rather than 0.8). Strictly speaking, is a **proper** fraction: the numerator is less than the denominator. An **improper** fraction has the numerator greater than the denominator. So is an improper fraction. An improper fraction can also be written as a 'mixed number', and it is important that you can convert an improper fraction to a mixed number and vice versa.

In the dim and distant past you may have written division sums like:

2 rem. 3

The GCSE version of this sum looks like this:

This does make sense if we think, say, of cakes. Imagine there are eleven quarter cakes. We can group eight of the quarters into two sets of four, to make two whole cakes. Then there are still three bits left. So there are two whole cakes and three-quarters of a cake.

The reverse process, to convert a mixed number to an improper fraction, is as follows.

The denominator will be 4. The numerator will be . All we are doing is thinking of the two whole cakes as eight quarters.

.

Modern calculators usually perform both of the above conversions. The fraction button is usually shown as . Calculators work with both types of fraction, but seem to prefer mixed numbers. Thus if you input as follows:

2 3 4 $\boxed{=}$

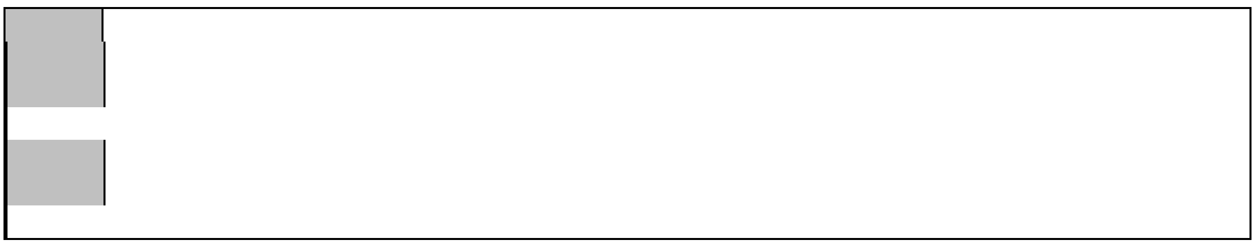
nothing happens: the fraction remains as , although the calculator's way of displaying this is as: 2 ┘3 ┘ 4.

However, if you now press SHIFT , the number displayed changes to 11 ┘ 4, which is the calculator's version of .

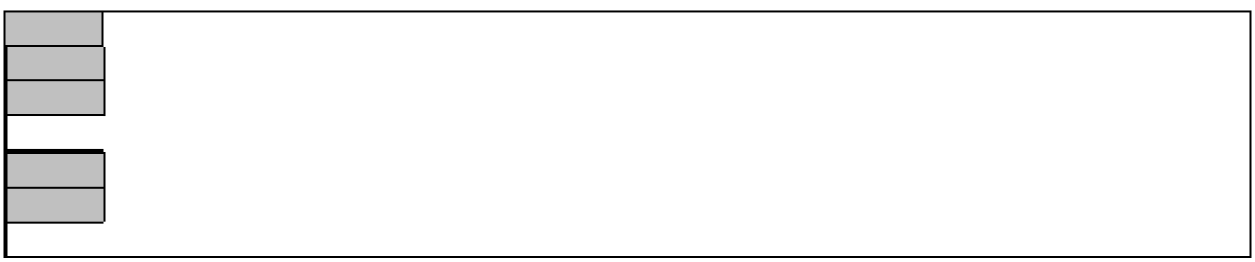
Equivalent Fractions

Believe it or not, the fraction is the same as : they are 'equivalent fractions'.

The following rectangle is divided into four equal sections, and then three of the sections are shaded:



Now take the same rectangle and divide it into eight equal sections, and then shade six of the sections:



The effect is the same. It can be seen from the diagrams that is the same as . In terms of arithmetic, we have shown that: .

Thus if we multiply top and bottom of a fraction by the same number, then the fraction has not changed.

The same is true for division: .

So we can divide top and bottom of a fraction by the same number without changing the fraction. This is called 'cancelling down' the fraction. We do this when we want to express a fraction in its 'lowest terms': this simply means that the numbers used are the smallest possible. Thus the fraction , expressed in its

‘lowest terms’ is .

In general, the working can be more complicated. The strategy is to find the largest number that divides into the top and bottom (i.e. the Highest Common Factor of the numerator and denominator), and then divide top and bottom by this HCF.

Example
Express the following fractions in their ‘lowest terms’:
(a) (b) (c)

(a) The HCF of 24 and 30 is 6. So divide top and bottom of the fraction by 6:

.

(b) The HCF of 75 and 100 is 25. Divide top and bottom by 25:

.

(c) The HCF of 7 and 12 is 1. So this is a trick question! The fraction is already in its ‘lowest terms’.

Most modern calculators will express a fraction in its ‘lowest terms’ automatically. If you enter a fraction and then press $\boxed{=}$ or $\boxed{\text{ENTER}}$, the calculator displays the fraction in its lowest terms. For example, if you do the following on a modern Casio:

24 30 $\boxed{=}$

the display becomes .

Why bother with ‘equivalent fractions’? This is a reasonable question. They are essential for adding and subtracting fractions (without a calculator).

Adding and Subtracting Fractions

Adding and subtracting is easy, but **only** when the denominators are the same! In this case, simply add or subtract the numerators:

For example: and .

However, if the denominators are different, we have to **make them the same**

before adding or subtracting.

Example 1

To add and , we can change the first fraction into quarters:

we have used ‘equivalent fractions’ to rewrite as . The top and bottom of the fraction have been multiplied by 2, so the fraction has not really changed.

But how did we know to work with quarters? The next two Examples develop the necessary ideas.

Example 2

This is harder. *Both* denominators have to be changed. One method that always works is to multiply the two denominators together. In this case, . So we make both denominators into 15. To do this we must multiply top and bottom of the first fraction by 5, and we must multiply top and bottom of the second fraction by 3. Note that this does not change the value of these fractions: it just presents them in a different way that makes addition possible.

Example 3

It is possible to multiply the denominators to get , and then work with a denominator of 24 in both fractions. However, this creates some unnecessary work. It is quicker and easier to work with a denominator of 12. Earlier in this Lesson you came across the idea of the Lowest Common Multiple of two numbers. That is precisely what we need here. 6 and 4 both divide into 12, and this is the smallest such number. We then need to multiply top and bottom of the first fraction by 2, and multiply the top and bottom of the second fraction by 3:

To summarise, addition or subtraction of fractions is performed by changing the denominator of each fraction into the ‘Lowest Common Denominator’.

‘Lowest Common Denominator’ is not a new idea: it is simply the Lowest Common Multiple of the denominators of the fractions.

Make sure you understand the following Example, which concentrates on the Lowest Common Denominator of the fractions, being the overall strategy rather than the full working of the sum.

Example 4

Do not work out the following additions/subtractions. Merely state the Lowest Common Denominator in each case:

(a) **(b)** **(c)**

(a) The Lowest Common Multiple of 10 and 5 is 10.
(b) The Lowest Common Multiple of 6 and 8 is 24.
(c) The Lowest Common Multiple of 13 and 5 is 65.

The method of the Lowest Common Denominator is also useful when trying to compare the sizes of fractions.

Example 5

Put the following lists of fractions in order of size, smallest first:

(a) , , (b) , , .

(a) The Lowest Common Multiple of 4, 16 and 8 is 16. So change the first of the fractions so that its denominators is 16: . Similarly, for the third fraction: . Now that the denominators are all the same, we simply compare the numerators, the smallest is 10, the middle is 12 and the largest is 13. The list of fractions in order of size, smallest first, is therefore: , , .

(b) The Lowest Common Multiple of 3, 4 and 5 is 60 (we simply multiply them together in this case). The first fraction can be written as: .

The second can be written as .

The third can be written as . Comparing the numerators, we see that the list is already in size order, smallest first.

Multiplying Fractions

The rule is fairly straightforward. We multiply numerators and denominators separately. However, it helps to simplify the arithmetic by 'cancelling down' first if possible; otherwise. simplification will be required later, and it might even be harder.

Example

(a) (b)

(a) No cancelling is possible, so multiply numerators and denominators separately:

(b) Cancelling is possible: 3 divides into both the three and the six:

Division of Fractions

The rule for dividing one fraction by another can be summarised simply:

1. Turn the second fraction upside down.
2. Then multiply the two fractions.

Example 1

(a) (b)

(a) Turn the second fraction upside down and multiply:

Note that the answer could be changed to the mixed number .

(b) Turn the second fraction upside down, and then multiply. But note that ‘cancelling’ is required before the multiplication:

Unfortunately, that is not the full story of arithmetic with fractions. Mixed numbers have not been considered. There are differing views on whether to perform certain operations with mixed numbers or improper fractions.

However, it is clear that multiplication and division are much simpler for improper fractions. So when confronted by multiplication or division of mixed numbers, it is best to convert to improper fractions first.

Example 2

(a) **(b)**

(a) Change both mixed numbers to improper fractions:

and .

Multiply: .

The answer could then be rewritten as the mixed number .

(b) Change both mixed numbers to improper fractions:

and .

Now invert the second fraction and multiply (note the cancelling):

.

The final answer could also be written as the mixed number .

Addition of mixed numbers is fairly easy: just add the whole numbers together, then add the fractions together. However, there might be adjustments to make.

Example 3
(a) (b) (c)

(a) Add the whole numbers: 1 + 2 = 3. Add the fractions (this is very easy, since the denominators are already the same): . The final answer is therefore quite simply .
(b) The whole numbers again add up to 3. But the fractions add up to . It is necessary to rewrite this as the mixed number .
We therefore conclude that .

(c) The whole numbers add up to 5. The fractions add up to:

The final answer is therefore .
Subtraction of mixed numbers can be similar to addition.
Example 4

Do the whole numbers first: 10 – 7 = 3.
Then do the fractions: .
Thus the final answer is .

However, subtraction of mixed numbers can be harder, involving a type of borrowing. If the whole numbers are small it is easier to work with improper fractions.

Example 5

The problem here is that is greater than , so that if we subtract the fractions we end up with a negative answer. It is possible to work round this. However, the recommended method is to convert to improper fractions:

The final answer could then be written as .

“Fraction of”

Last but not least is how to find a fraction of a quantity.

Example

Find (a) of £40 (b) of

(a) One fifth of £40 is £8 (since). We therefore require £24 (since).

Alternatively, “of” really means multiply, and we can think of 40 as the fraction , so , so that the answer is again £24.

(b) Simply replace the “of” by multiply ():

Fractions within Fractions

When fractions occur within fractions, e.g.

,

the numerator and denominator of the whole fraction must be evaluated separately before the final answer is worked out. Here,

Therefore,

With other questions, the order given in the previous section must be observed, e.g.

.

First work out :

Back to the problem:

Answer the following questions without using a calculator.

•

1. Convert the following mixed numbers to improper fractions:

(a) (b) (c) (d)

2. Convert the following improper fractions to mixed numbers:

(a) (b) (c) (d)

3. Express the following fractions in their lowest terms:

(a) (b) (c) (d)

4. Rewrite the following list of fractions in order of size, smallest first:

5. Rewrite the following list of fractions in order of size, largest first:

For Q.6-9, leave your answers as fractions in their lowest terms.

6. Perform the following additions, writing your answer as a fraction in its lowest terms:

(a) (b) (c) (d)

(e) (f)

7. Perform the following subtractions, writing your answer as a fraction in its lowest terms:

(a) (b) (c) (d)

(e) (f)

8. Perform the following multiplications, writing your answer as a fraction in its lowest terms:

(a) (b) (c) (d)

(e)

9. Perform the following divisions, writing your answer as a fraction in its lowest terms:

(a) (b) (c) (d)

10. Find the following, writing your answer as a fraction in its lowest terms:

(a) of 90 (b) of 24 (c) of 60.

Inverse Operations

(An **operation** is a mathematical action or event, e.g. the act of multiplying 4 by 4.)

If three hens produce five eggs each, how many eggs do we have altogether? 3 5 = 15. But sometimes it's useful to start from the other end. Say we have 15 eggs and we know they were produced equally by 3 hens: how many eggs did they produce each? 15 3 = 5 eggs each. The two sums:

3 5 = 15 and 15 3 = 5

are very similar except the numbers are in different positions and the operator (or) has been changed (or reversed). We could also write 3 = 15 5 or 15 = 3 5 or even 15 = 5 3. But we must be very careful when moving a number from one side of the equals sign to the other.

An inverse operation is an operation which *undoes* another.
Here are some examples:

4 + 7 = 11 4 = 11 – 7

(here the inverse of the operation +7 is –7 because you end up back where you started, i.e. with 4)

3 5 = 15 3 = 15 5
12 4 = 3 12 = 3 4

You may have to imagine a + sign before the first number, so we also have:

4 + 7 = 11 7 = 11 – 4
3 5 = 15 5 = 15 3

but not

12 4 = 3   4   = 3 12

Remember that the two sides of a completed sum (or **equation**, as you will learn to call it) can swap over *in their entirety* without having to worry about signs, e.g.

4 + 3 = 7   7 = 4 + 3
3 5 = 15   15 = 3 5

Suggested Answers to Activities

Activity One

1. $100 = 10^2$, $121 = 11^2$, $144 = 12^2$, $169 = 13^2$, $196 = 14^2$

2. 53, 59, 61, 67

3. (a) 3
(b) 1
(c) 8

4. (a) 24
(b) 50
(c) 84

5. (a) 1, 3, 5, 9, 15, 45
(b) 1, 2, 23 46
(c) 1, 47

6. 37

7. 59, 61
8. (a) 22   (b) 13 or 29 or 47   (c) 45   (d) 21   (e) 49

9. (a)
(b)
(c)

Activity Two

(a) $3 + 2 \times 8$
$= 3 + (2 \times 8)$
$= 3 + 16$

$= 19$

(b) $4 \times 5 - 3$
$= (4 \times 5) - 3$
$= 20 - 3$
$= 17$

(c) $9 \times 2 + 64 \div 8$
$= (9 \times 2) + (64 \div 8)$
$= 18 + 8$
$= 26$

(d) $25 \div 5 - 9 \div 3$
$= (25 \div 5) - (9 \div 3)$
$= 5 - 3$
$= 2$

(e) $81 \div (5 + 4) - 2 \times 3 + 14 \div 7$
$= (81 \div 9) - (2 \times 3) + (14 \div 7)$
$= 9 - 6 + 2$
$= 5$

(f) $17 + 9 - 14 + 5 - 20 + 1 - 2$
$= 17+ 9 + 5 + 1 - 14 - 20 - 2$
$= 32 - 36$
$= - 4$

(g)

Activity Three

1. (a) 16, 18 (b) 45, 54 (c) 2, 6 (d) 5, 2
(e) 320, 640 (f) 160, 32 (g) 42, 68 (h) 34, 55

2. (a) 19 (b) -6 (c) 55 (d) 16
(e) 81 (f) 13

3. (a) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
(b) (i) 82, 101 (ii) 810, 1000 (iii) 80, 99

4. (a) 45, 66 (b) 47, 65

Activity Four

1. (a) (b) (c) (d)

2. (a) (b) (c) (d)

3. (a) (b) (c) (d)

4.

5.

6. (a) (b) (c)
(d) (e) 4 (f)

7. (a) (b) (c)
(d) (e) (f)

8. (a) (b) (c)
(d) (e) 14

9. (a) (b) (c) (d)

10. (a) 60 (b) 18 (c) 42